Dynamic Cross Hedging with Mortgage-Backed Securities

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Dealers and portfolio managers tend to hold significant positions in mortgage-backed securities (MBS) commonly known as passthroughs. Adverse movements in interest rates significantly affect the market value of these securities. It is therefore no surprise that hedging interest rate risk for MBS has been a topic of intense theoretical and empirical investigation. The large financial losses incurred in 1993 and 1994 have renewed the debate on the effectiveness of commonly used hedging strategies.

A fixed-rate mortgage-backed security is essentially a bond that is backed by a pool of fixed-rate mortgage loans. Principal and interest payments of the underlying pool are "passed through" to the investor. While principal and interest payments are default-free, due to an implicit government guarantee, the timing of the cash flows is highly uncertain because of the prepayment option that allows the mortgagor to prepay part or all of the mortgage at any time.[1]

The prepayment option renters the duration of MBS indeterminate. As a result, the impact of interest rate changes on the price of MBS is difficult to ascertain. As pointed out by Breeden [1997], prepayments induce asymmetry in mortgage returns because losses due to rate increases are larger than gains due to rate reductions.

Following the demise of the GNMA collateralized depository receipt (CDR) futures contract in 1987 and the mortgage-backed futures contract (MBF) in 1992, cross-hedging with Treasury futures has become the most commonly used approach by investors who want to hedge the price risk of MBS.[2] As pointed out by Anderson and Danthine [1981], most hedging decisions actually involve some sort of cross-hedging because the asset (commodity) for delivery in the futures contract differs from the spot asset held.

The most popular cross-hedging instrument for MBS has been the ten-year Treasury note futures contract because the effective duration of the typical MBS is closer to the duration of the ten-year Treasury note than to that of any other government security. Fernald, Keane, and Mosser [1994] report that hedging of mortgage-backed securities with Treasury note futures has become so intense in the last few years that the short-run dynamics of the term structure of interest rates have changed.

Commonly used hedging techniques are based on 1) empirical hedge ratios, and 2) valuation model-based hedge ratios. The optimal hedge ratio according to the former approach is the slope coefficient from a regression of spot price changes on futures price changes. Intuitively, the estimated hedge ratio measures the average responsiveness of spot price changes to futures price changes. This method yields time-invariant variance-minimizing ratios (see Bera, Bubnys, and Park [1993], Ederington [1979], and Park and Bera [1987], among others).

The valuation model-based approach focuses on matching the duration of the MBS to the duration of a Treasury security in order to neutralize interest rate risk. There are, however, serious difficulties in devising accurate model-based duration measures for MBS (see Babbel and Zenios [1992], Barlett [1994], and Davidson and Herskovitz [1994], among others). These difficulties are related to assumptions about interest rate movements and prepayment rates that constitute an integral part of all valuation models of MBS. Breeden [1994] finds that prepayment rates are highly correlated with the difference between coupon rate and refinancing rate.

The estimation of optimal hedging ratios using regression analysis is not free of problems either. First, if the prices of MBS are cointegrated with the prices of the Treasury futures (i.e., the hedging instrument), standard ordinary least squares methods will tend to overdifference the data and obscure the long-term relationship between the MBS prices and the Treasury futures prices. As a result, the estimated hedging ratios will be downward biased (see, for example, Brenner and Kroner [1995], Engle and Granger [1987], and Kroner and Sultan [1993]).

Second, regression analysis assumes that the joint distribution of price changes in the spot and futures markets can be characterized by time-invariant first and second moments. Consequently, the risk hedging ratio is treated as time-invariant. Such an assumption, however, runs contrary to the findings of Anderson [1985], Kroner and Sultan [1993], and Malliaris and Urrutia [1991] among others.

To deal with this problem, Breeden [1991 and 1994] proposes a dynamic hedging strategy based on market prices for MBS with different coupons, the so-called roll-up, roll-down approach (RURD). He finds that during the volatile period 1982-1986, dynamic hedge ratios based on the RURD method provided better results than the traditional static hedge ratios.

Studies using regression-based approaches find that hedge ratios based on time-varying second moments produce greater risk reduction than constant covariance models. See, for example, Baillie and Myers [1991] and Myers [1991] for commodities; Cecchetti, Cumby, and Figlewski [1988] for Treasury bonds; Gagnon and Lypny [1995] for Canadian banker's acceptances; Kroner and Sultan [1993] for foreign currency; and Park and Switzer [1995] for stock index futures.

Despite the evidence from these markets that hedge ratios based on time-varying second moments can be a significant improvement over the traditional static hedging techniques, no empirical work has been undertaken to evaluate their applicability in a very important market, namely, the mortgage-backed securities market. This article proposes a dynamic hedging strategy for fixed-rate mortgage-backed securities based on the time-varying covariance structure of the joint distribution of price changes of Federal National Mortgage Association MBS and Treasury note futures. Using daily data, dynamic hedge ratios are estimated from a bivariate error correction model with a GARCH error structure. This model has the advantage that it preserves the long-term equilibrium between the two markets while allowing short-term dynamics to be characterized by a time-varying variance/covariance structure.

The hedging effectiveness of the dynamic hedge ratios obtained from this model is compared to several alternative hedging techniques. The comparison is carried out in terms of maximum risk reduction as well as expected utility maximization. The findings show that hedge ratios that are based on the error correction GARCH model are, within-sample and out-of-sample, superior to those derived from models that assume a constant variance/covariance structure. Accounting for transaction costs due to rebalancing does not alter the conclusions.

I. DATA AND PRELIMINARY STATISTICS

The data used in this study include daily closing spot prices for the thirty-year FNMA securities with coupons of 7.5%, 8%, 8.5%, 9%, 9.5%, and 10% compiled by Smith Barney, Inc. Data used for the futures market are the daily settle prices for the ten-year Treasury note futures contracts, as obtained from Tick Data, Inc. The delivery months for the ten-year Treasury note futures contract are March, June, September, and December. On any given trading day, there are prices for all four outstanding contracts. A time series of non-overlapping observations is created by using the most actively traded contract, thus avoiding problems related to thin trading. The sample extends from July 21, 1992, to August 14, 1995, and includes a total of 761 daily prices and 760 daily returns for each series.

The time series properties of the various-coupon thirty-year FNMA-MBS and the ten-year Treasury futures are important in choosing the best model specification. The prices of most financial assets, or, their logarithmic transformations, have been found to be non-stationary; i.e., they have a unit root in their univariate representation (see Brenner and Kroner [1995]).

Exhibit 1, Panel A, reports the results from unit root tests based on the augmented Dickey-Fuller [1979 and 1981] test (ADF) for the logarithms of the prices of the MBS and the Treasury note futures contract. The calculated statistics are in all instances below the 5% critical value.[3]

It follows, therefore, that the logarithms of the prices have a unit root, and first differencing is both necessary and sufficient to induce stationarity. As expected, the Dickey-Fuller tests reject the hypothesis of a unit root in the first logarithmic differences (returns).[4] If the prices of MBS and Treasury note futures share a common stochastic trend, however (that is, are cointegrated), then modeling of the first logarithmic differences should incorporate an error correction term imposing the long-term equilibrium that is implied by the common stochastic trend (see Engle and Granger [1987] and Engle and Yo [1987]).

The cointegration test proposed in Engle and Granger [1987] is essentially a test for a unit root applied to the residuals of a regression involving the variables to be tested for a common stochastic trend. The notion is that if the linear combination implied by the estimated regression is stationary, then the variables are cointegrated.

As can be seen from Exhibit 1, Panel A, the estimated Engle-Granger (EG) statistic suggests that the logs of the prices of the FNMAs with coupons of 7.5%, 8%, 9%, and 10% are cointegrated with the logs of the prices of the Treasury futures, while the FNMAs with coupons of 8.5% and 9.5% are not. Thus, in at least four of the six MBS examined, modeling of the short-term conditional means requires the inclusion of an error correction term based on the cointegrating regression.

Exhibit 1, Panel B, reports several descriptive statistics for the percentage logarithmic differences of the prices, referred to as returns henceforth, of the FNMA securities and the ten-year Treasury futures. These are the sample means and standard deviations, measures of skewness and excess kurtosis, and Ljung-Box statistics for up to twenty lags.

In all cases, the daily mean returns of the series under study are statistically insignificant. The Kolmogorov-Smirnov (D) statistic fails to reject normality in only two cases, namely, the 7.5% FNMA and the Treasury futures. The skewness and excess kurtosis mea-

EXHIBIT 1
Preliminary Statistics

	FNMA 7.5	FNMA 8	FNMA 8.5	FNMA 9	FNMA 9.5	FNMA 10	10-year Treasury Futures
Panel A. Unit Root and Cointegration Tests							
ADF	−1.49	−1.52	−1.61	1.94	−1.96	−1.51	−1.43
ADF (differenced)	−17.33*	−17.68*	−17.51*	−17.68*	−17.18*	−17.80*	−16.35*
EG	−4.43*	−3.52*	−3.09	−3.53*	−2.73	−4.78*	
Panel B. Summary Statistics							
Mean	−0.00004	−0.0137	−0.00246	−0.00286	−0.00267	0.00049	0.00252
Standard Deviation	0.34021	0.29142	0.23884	0.19725	0.16484	0.20248	0.41798
Skewness	−0.65991*	−0.69558*	−0.31140*	−0.53430*	0.00049	1.50224*	−0.25643*
Kurtosis	4.59586*	5.25111*	2.39282*	3.61365*	1.50224*	99.23884*	1.61190*
K-S Statistics	0.04750	0.05330*	0.06090*	0.06740*	0.07070*	0.19340*	0.04350
LB(20) for $r_{j,t}$	27.9691	32.6269*	34.4814*	52.1197*	37.9553*	78.6152*	29.8054
LB(20) for $r^2_{j,t}$	64.5213	62.9872*	169.1967*	138.9765*	101.4164*	171.2007*	18.9934

*Statistically significant at the 5% level.

sures are statistically significant, indicating departures from normality in all instances.

Non-normality can be caused, at least partially, by temporal dependencies in the return series, especially second-moment temporal dependencies. The presence of such dependencies is tested by means of the Ljung-Box (LB) statistic.[5]

The hypothesis that all autocorrelations up to the twentieth lag are jointly zero is rejected in most cases (exceptions are the 7.5% FNMA and the Treasury note futures). Thus, first-moment temporal dependencies are present. The hypothesis that the autocorrelations for the squared returns up to the twentieth lag are jointly zero is rejected across all FNMA returns but retained for futures returns.

It is interesting to note that the LB statistics are much higher for the squared returns. This provides indirect evidence of time-varying second moments and justification for the subsequent use of the bivariate GARCH specification for the variance/covariance matrix.

II. METHODOLOGY

Conventional hedging strategies are based on the assumption that the hedger's objective is risk minimization: see Ederington [1979], Johnson [1960], Malliaris and Urrutia [1991], and Stein [1961], among others. The optimal hedge ratio is then found by regressing the returns (or price changes) of a cash position on the returns (or price changes) of the hedging instrument:

$$S_t - S_{t-1} = \alpha + \beta(F_t - F_{t-1}) + \varepsilon_t \tag{1}$$

where S_t is the spot price (or the logarithm of the price) of the asset to be hedged at time t; F_t is the price (or the logarithm of the price) of the hedging instrument during period t; and ε_t is the random error term assumed to be an i.i.d. process.[6] The slope coefficient, β, is known as the optimal hedge ratio and defines the number of hedging instruments (usually futures) to go short per unit of the asset held in the spot market. Under the assumption that futures prices follow a martingale process, the minimum-variance hedge ratio coincides with the optimal hedge ratio for an expected utility-maximizing agent with quadratic utility function.

Several aspects of this approach are problematic. First, the assumption that the variance-covariance matrix of the joint distribution of spot and futures returns is constant is most likely violated. Second, cointegration between the prices of the spot asset and the futures implies certain restrictions in the modeling of the conditional means. Our objective is to analyze the incremental benefit from using dynamic (time-varying hedges) as opposed to static hedges for MBS.

Following Baillie and Myers [1991], Cecchetti, Cumby and Figlewski [1988], and Kroner and Sultan [1993], we denote the return (percentage price changes) of a share of an MBS by $r_{s,t}$ and the return of the futures hedging instrument by $r_{f,t}$. A hedged portfolio with a short position of h_{t-1} dollars worth of futures for every dollar worth of the spot asset held at time $t - 1$ will have a random return $r_{h,t}$ with conditional mean and variance given by:

$$E_{t-1}(r_{h,t}) = E_{t-1}(r_{s,t}) - h_{t-1}E_{t-1}(r_{f,t}) \tag{2}$$

$$\begin{aligned} Var_{t-1}(r_{h,t}) = {} & Var_{t-1}(r_{s,t}) + h^2_{t-1}\, Var_{t-1}(r_{f,t}) - \\ & 2h_{t-1}Cov_{t-1}(r_{s,t}, r_{f,t}) \end{aligned} \tag{3}$$

Assuming a time-separable quadratic utility function, the optimal hedge ratio h_{t-1} will be such that expected utility as of time $t - 1$ is maximized, or:

$$\max_{h_{t-1}} E_{t-1}U(r_{h,t}) = \max\, [E_{t-1}(r_{h,t}) - \theta Var_{t-1}(r_{h,t})] \tag{4}$$

where U(.) is the utility from holding the hedged portfolio and $\theta > 0$ is the coefficient of risk aversion. The first-order condition for a maximum yields the optimal hedge ratio:

$$\begin{aligned} h^*_{t-1} = {} & Cov_{t-1}(r_{s,t}, r_{f,t})/Var_{t-1}(r_{f,t}) - \\ & E_{t-1}(r_{f,t})/2\theta Var_{t-1}(r_{f,t}) \end{aligned} \tag{5}$$

As can be seen, the optimal hedge ratio has two distinct components. The first component is the conditional variance-minimizing hedge ratio, and the second component is the speculative demand for futures contracts. The expected utility-maximizing hedge ratio coincides with the variance-minimizing ratio if 1) the coefficient of risk aversion is extremely high, and/or 2) the expected percentage change in the futures price is zero.

The statistical insignificance of the sample percentage change of the futures price lends support to the notion that futures prices follow a martingale process without drift. Consequently, the second component of the optimal hedge ratio vanishes, and the expected utility-maximizing ratio coincides with the variance-minimizing hedge ratio. Baillie and Myers [1991], Gagnon and Lypny [1995], Kroner and Sultan [1993], and Park and Bera [1987], among others, adopt a similar approach.

Estimation of the dynamic hedge ratios over time based on (5) can be obtained provided the conditional joint distribution of the spot and futures prices is fully specified. On the basis of the preliminary findings that futures and spot prices are cointegrated, the percentage price changes are modeled as a bivariate error correction process with time-varying variance/covariance matrix. The model is described by the set of equations:

$$r_{s,t} = \beta_{s,0} + \beta_{s,1}(S_{t-1} - \delta_0 - \delta_1 F_{t-1}) + \varepsilon_{s,t} \qquad (6)$$

$$r_{f,t} = \beta_{f,0} + \beta_{f,1}(S_{t-1} - \delta_0 - \delta_1 F_{t-1}) + \varepsilon_{f,t} \qquad (7)$$

$$\sigma^2_{s,t} = \alpha_{s,0} + \alpha_{s,1}\varepsilon^2_{s,t-1} + \alpha_{s,2}\sigma^2_{s,t-1} \qquad (8)$$

$$\sigma^2_{f,t} = \alpha_{f,0} + \alpha_{f,1}\varepsilon^2_{f,t-1} + \alpha_{f,2}\sigma^2_{f,t-1} \qquad (9)$$

$$\sigma_{s,f,t} = \gamma_0 + \gamma_1\varepsilon_{s,t-1}1\varepsilon_{f,t-1} + \gamma_2\sigma_{s,f,t-1} \qquad (10)$$

where $r_{j,t}$ is the return at time t for j = s, f (s = spot, f = futures), $\varepsilon_{j,t}$ is the innovation (error term), $\sigma^2_{j,t} = \mathrm{Var}(\varepsilon_{j,t}|\Omega_{t-1})$ is the conditional variance, and $\sigma_{s,f,t} = \mathrm{Cov}(\varepsilon_{s,t}\,\varepsilon_{f,t}\,|\,\Omega_{t-1})$ is the conditional covariance between the returns of the MBS and the ten-year Treasury futures. Equations (6) and (7) model the conditional means of the spot and futures returns respectively. The error correction term $(S_{t-1} - \delta_0 - \delta_1 F_{t-1})$ is a measure of the extent to which the system is out of equilibrium. S_t and F_t are the logarithms of the spot and future prices, respectively.

The conditional variance/covariance structure described by (8)-(10) implies that each variance is a function of own past values and past squared errors, while the covariance is a function of own past values and past cross-residuals, e.g., Bollerslev, Engle, and Wooldridge [1988]. Assuming conditional normality, the log likelihood function of the bivariate error correction GARCH model can be written as:

$$L(\Theta) = -T\log(2\pi) - (1/2)\sum_{t=1}^{T}(\log|H_t| + \varepsilon t H_t^{-1}\varepsilon_t') \qquad (11)$$

where Θ is the 13 × 1 parameter vector to be estimated, H_t is the 2 × 2 time-varying conditional variance-covariance matrix with diagonal elements given by Equations (8) and (9) and off-diagonal elements given by (10), $[e_{s,t}, e_{f,t}]$ is the 1 × 2 vector of innovations at time t. Due to the non-linearity of the log-likelihood function, numerical maximization techniques based on the Berndt, Hall, and Hausman [1974] algorithm are used to obtain estimates of the parameter vector.

III. HEDGING PERFORMANCE AND COMPARISONS

The model is estimated for six different-coupon FNMA MBS. The hedging instrument used in all instances is the ten-year Treasury note futures contract.

Model Estimates

Exhibit 2, Panel A, reports the maximum likelihood estimates of the bivariate error correction GARCH (EC-GARCH) model described by Equations (6)-(10), using data on FNMA MBS with coupons of 7.5%, 8.0%, 8.5%, 9%, 9.5%, and 10%. The coefficients describing the conditional variance/covariance equations are statistically significant at any level of significance. Thus, the conditional variances of all MBS depend on last-period's squared innovations and last-period's conditional variances.

Volatility persistence is high across coupons, with the exception of the 10% coupon. The implication is that shocks to volatility "die out" very slowly.

The covariances between the returns of different-coupon FNMAs and the Treasury note futures are time-varying. Specifically, they depend on the cross-product of past innovations and past covariances. It should be noted that the correlations implied by Equations (8)-(10) are also time-varying.

A constant correlation version along the lines of Kroner and Sultan [1993] and Park and Switzer [1995] was also estimated. In all cases, the constant correlation hypothesis was rejected. Such rejections are highly likely in situations of cross-hedging as opposed to direct hedging.

Correct specification of the time-varying variance/covariance matrix requires that the estimated squared standardized residuals and their cross-products follow i.i.d. processes. To test this hypothesis, the LB statistic is used. The estimated values of the LB statistic, using twenty lags, are statistically insignificant for the

EXHIBIT 2
Maximum Likelihood Estimates

	FNMA 7.5	FNMA 8	FNMA 8.5	FNMA 9	FNMA 9.5	FNMA 10
Panel A. Bivariate EC-GARCH Estimates						
$\beta_{s,0}$	−0.0037	−0.0001	0.0004	0.0015	0.0003	0.0150
	(0.0115)	(0.0087)	(0.0062)	(0.0043)	(0.0048)	(0.0036)*
$\beta_{f,0}$	−0.0006	0.0059	0.0116	0.0159	0.0142	0.0306
	(0.0151)	(0.0130)	(0.0121)	(0.0118)	(0.0127)	(0.0139)*
$\beta_{s,1}$	−0.0675	0.1348	−0.4418	−0.3066	−0.6195	−4.9499
	(1.6406)	(0.8534)	(0.6046)	(0.5550)	(0.5555)	(0.6173)*
$\beta_{f,1}$	4.8728	3.1171	1.6640	1.2931	−0.4279	−3.4337
	(2.3308)*	(1.4789)*	(1.2737)	(1.4697)	(1.6405)	(2.6728)
$\alpha_{s,0}$	0.0023	0.0005	0.0002	0.0001	0.0002	0.0013
	(0.0002)*	(0.0001)*	(0.0001)*	(0.0000)*	(0.0000)*	(0.0002)*
$\alpha_{f,0}$	0.0050	0.0024	0.0039	0.0037	0.0061	0.0104
	(0.0007)*	(0.0004)*	(0.0006)*	(0.0007)*	(0.0012)*	(0.0025)*
$\alpha_{s,1}$	0.0216	0.0375	0.0443	0.0690	0.0512	0.2669
	(0.0024)*	(0.0034)*	(0.0044)*	(0.0062)*	(0.0063)*	(0.0165)*
$\alpha_{f,1}$	0.0123	0.0269	0.0245	0.0333	0.0351	0.0650
	(0.0030)*	(0.0051)	(0.0050)*	(0.0073)	(0.0077)*	(0.0158)*
$\alpha_{s,2}$	0.9567	0.9559	0.9495	0.9292	0.9387	0.7384
	(0.0031)*	(0.0033)*	(0.0045)*	(0.0065)*	(0.0072)*	(0.0187)*
$\alpha_{f,2}$	0.9593	0.9601	0.9516	0.9452	0.9283	0.8808
	(0.0052)*	(0.0046)*	(0.0054)*	(0.0086)*	(0.0108)	(0.0226)*
γ_0	0.0020	0.0006	0.0004	0.0003	0.0003	0.0014
	(0.0002)*	(0.0002)*	(0.0001)*	(0.0001)*	(0.0001)*	(0.0004)*
γ_1	0.0138	0.0294	0.0302	0.0459	0.0355	0.1219
	(0.0018)*	(0.0034)*	(0.0033)*	(0.0058)*	(0.0046)*	(0.0173)*
γ_2	0.9695	0.9644	0.9627	0.9472	0.9552	0.8428
	0.0018*	(0.0028)*	(0.0029)*	(0.0062)*	(0.0055)*	(0.0187)*
Panel B. Model Diagnostics						
LB(20) for $z^2_{s,t}$	12.9151	13.1150	18.7322	10.0106	6.6978	2.5592
LB(20) for $z^2_{f,t}$	14.3369	10.9230	9.6887	9.0498	8.7855	11.4058
LB(20) for $z_{s,t}z_{f,t}$	23.7598	21.7170	33.2295*	15.9229	19.6967	15.2473
K-S for $z_{s,t}$	0.0419	0.0456	0.0451	0.0459	0.0557*	0.0382
K-S for $z_{f,t}$	0.0405	0.0431	0.0417	0.0424	0.0419	0.0429

	LL	LR	LL	LR	LL	LR	LL	LR	LL	LR	LL	LR
Panel C. Tests on Model Restrictions												
EC-GARCH	1307		1383		1501		1583		1608		1521	
GARCH	1300	14	1379	8	1498	6	1581	4	1607	2	1502	38
EC-OLS	1268	78	1296	174	1370	262	1449	268	1512	192	1214	614
OLS	1260	94	1260	246	1292	418	1367	432	1509	198	1197	648

Standard errors in parentheses.
*Statistically significant at the 5% level at least.

squared standardized residuals across all six FNMA MBS under investigation (Exhibit 2, Panel B). Consequently, the hypothesis that all autocorrelations up to the twentieth lag are jointly zero is retained. The results for the cross-products are similar in the sense that all LB statistics, with the exception of the 8.5% coupon, are statistically insignificant.

Overall, the LB statistics show that the models are well specified. Equally important, the assumed density function is not rejected on the basis of the Kolmogorov-Smirnov statistic, with the only exception being the 9.5% coupon.

Exhibit 2, Panel C, reports log-likelihood values and likelihood ratio test statistics for various restrictions imposed on the coefficients of the full model (EC-GARCH). The restriction that the error correction term is insignificant is rejected for coupons 7.5%, 8%, 8.5%, and 10% but retained for coupons 9% and 9.5%. The hypothesis that the parameters describing the time-varying component of the variance-covariance matrix are jointly zero, i.e., $\alpha_{s,1} = \alpha_{s,2} = \alpha_{f,1} = \alpha_{f,2} = \gamma_1 = \gamma_2 = 0$, is rejected at any level of significance and across all coupons. Rejection of this hypothesis provides strong evidence that the second moments of FNMA MBS are time varying. Therefore, time-varying hedge ratios should be an improvement over traditional static hedge ratios. Finally, the classic OLS model is considered by testing the restriction that $\beta_{s,1} = \beta_{f,1} = \alpha_{s,1} = \alpha_{s,2} = \alpha_{f,1} = \alpha_{f,2} = \gamma_1 = \gamma_2 = 0$. Again, the restriction is rejected across coupons at any level of significance.

Within-Sample and Out-of-Sample Hedging Performance

Even though the EC-OLS and the OLS models are rejected, we contrast their hedging effectiveness to that of the full (EC-GARCH) model within-sample and out-of-sample. The reason is that the full model will probably require frequent revisions of the hedge ratio. If transaction costs are assumed to be present, such frequent revisions may not be economically feasible, so a simpler model may be preferable.

The time-varying optimal hedge ratios given in (5) are calculated using the estimated time-varying variance-covariance matrix based on the full bivariate EC-GARCH as well as various restricted versions. The evidence concerning stationarity of hedge ratios based on bivariate GARCH models is mixed.

Depending on the spot asset in question, the hedge ratios may follow random walks (see Baillie and Myers [1991] for commodities) or a stationary process (see Kroner and Sultan [1993] for exchange rates). Using the augmented Dickey Fuller (ADF), we fail to reject the hypothesis that hedge ratios for FNMA MBS based on the unrestricted EC-GARCH model follow random walks. Moreover, this is true across all coupons (see Exhibit 3, panel A). First differences, on the other hand, appear to be stationary.

Exhibit 3, Panel B, reports the minimum, maximum, mean, and standard deviation of within-sample estimated hedge ratios for all six coupons, even though

EXHIBIT 3

Summary Statistics and Time Series Properties of the EC-GARCH Hedge Ratios

	FNMA 7.5 HR	FNMA 8 HR	FNMA 8.5 HR	FNMA 9 HR	FNMA 9.5 HR	FNMA 10 HR
Panel A. Unit Root Tests						
ADF	−2.76	−1.53	−1.08	−1.70	−1.48	−1.76
ADF (differenced)	−33.59*	−31.19*	−32.94*	−34.50*	−28.12*	−24.42*
Panel B. Statistics for Hedge Ratios						
Min	0.3462	0.1900	0.1078	0.0691	0.0445	0.0055
Max	0.8818	0.7985	0.6707	0.6261	0.4865	0.4143
Mean	0.6720	0.5318	0.4095	0.3152	0.2418	0.2023
Standard Dev.	0.1004	0.1774	0.1746	0.1609	0.1210	0.1436

*Statistically significant at the 5% level at least.

EXHIBIT 4

Variance Comparisons of Within-Sample Hedging Effectiveness

Type of Hedge	FNMA 7.5	FNMA 8	FNMA 8.5	FNMA 9	FNMA 9.5	FNMA 10	Average
Panel A. Portfolio Variances							
EC-GARCH	0.0264	0.0219	0.0167	0.0140	0.0123	0.0310	
GARCH	0.0265	0.0219	0.0168	0.0140	0.0124	0.0330	
EC-OLS	0.0280	0.0257	0.0211	0.0171	0.0145	0.0331	
OLS	0.0284	0.0258	0.0212	0.0172	0.0146	0.0340	
Unhedged	0.1153	0.0847	0.0568	0.0389	0.0272	0.0411	
Panel B. Percentage Variance Improvement of Conditional Hedge Compared to:							
GARCH	0.38%	0.00%	0.60%	0.00%	0.81%	6.06%	1.31%
EC-OLS	5.71%	14.78%	20.85%	18.12%	15.17%	6.34%	13.49%
OLS	7.04%	15.12%	21.23%	18.60%	15.76%	8.82%	14.43%
Unhedged	77.10%	74.14%	70.60%	64.01%	54.78%	24.39%	60.83%

non-stationarity makes the interpretation of these statistics rather difficult. Still, it is interesting to see that, across coupons, the average hedge ratio never exceeds 0.672 (for the 7.5%), and it goes as low as 0.202 (for the 10%). In fact, there is a monotonic inverse relationship between coupon rates and hedge ratios.

This makes intuitive sense because the higher the coupon, the lower the duration, and hence the lower the price sensitivity of the spot asset (see also Breeden [1991 and 1994]). The optimal hedge ratio is thus lower. The maximum estimated hedge ratio is 0.882 (for the 7.5% coupon), and the minimum is 0.006 for the 10%.

The hedging effectiveness of the full model and its several restricted versions are evaluated in terms of variance reduction in relation to the unhedged position. Both within-sample and out-of sample evaluations are performed. Within-sample evaluations are based on hedge ratios implied by each particular model estimated over the entire sample period; i.e., we evaluate $Var(r_{s,t} - h_{t-1}r_{f,t})$, where $r_{s,t}$ and $r_{f,t}$ are the returns (percentage price change) on the spot asset and the futures contract, respectively, and h_{t-1} is the hedge ratio as of time $t - 1$.

The results are reported in Exhibit 4. Panel A reports the calculated variances based on the full model and three of its restricted versions. It is apparent that the least variance is achieved when the EC-GARCH model is used. Next in order come the GARCH model, the EC-OLS model, and finally the traditional OLS model. This monotonic behavior of the estimated variances is preserved across all six coupon FNMA MBS.

Panel B reports percentage reductions in variance using hedges based on the EC-GARCH model, the EC-OLS model, the standard OLS model, and finally the unhedged position. The percentage reduction in variance is essentially the hedging effectiveness measure suggested by Ederington [1979].

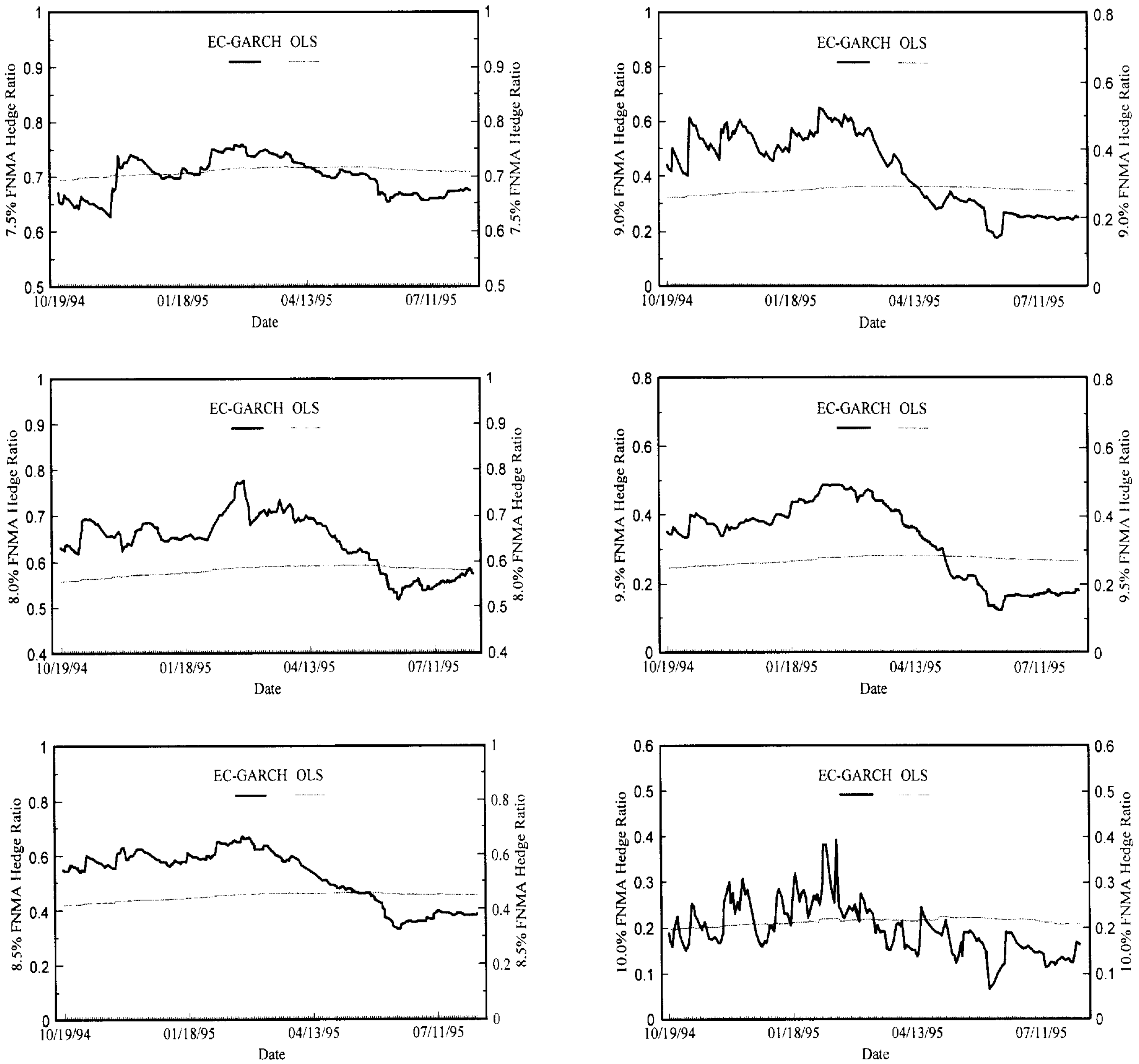
Compared to the unhedged position, the bivariate EC-GARCH model produces on the average a 60.83% reduction in variance with a minimum reduction of 24.39% (for the 10% coupon) and a maximum reduction of 77.10% (for the 7.5% coupon). Compared to the traditional OLS hedging approach, the average percentage variance reduction across coupons is 14.43% with a maximum reduction of 21.23% (for the 8.5% coupon) and a minimum reduction of 7.04% (for the 7.5% coupon).

Without considering transaction costs, it would appear that the superior hedging performance of the EC-GARCH model more than justifies the costs arising from using such a complex model. Studies dealing with other financial assets find smaller improvements in performance of using time-varying hedge ratios. For example, Kroner and Sultan [1993], using five different exchange rates, find an average improvement of 2.5% in terms of variance reduction over and above the traditional OLS hedge.

Historical (ex post) performance, however, is not the safest way to evaluate the usefulness of hedging models. Out-of-sample evaluation is an alternative and perhaps a more meaningful way of judging a model's validity and usefulness. To this end, the EC-GARCH model and the restricted models are reestimated using the first 560 observations and a holdout sample of 200 observations. At each time period the one-day-ahead hedge ratios are calculated and the models are reestimated by adding one more observation. This process is repeated until the holdout sample is depleted.

Exhibit 5 presents a graphical representation of the estimated one-day-ahead hedge ratios based on the EC-GARCH and the traditional OLS models. The EC-dynamic hedge ratios are clearly time-varying, while the OLS-based hedge ratios are nearly constant. The inverse relationship between coupon size and average hedge

EXHIBIT 5
Out-of-Sample Hedge Ratios for Thirty-Year FNMA



ratios is apparent in Exhibit 5.

Exhibit 6 Panel A, reports out-of-sample variances of hedged portfolios with hedge ratios based on the EC-GARCH, the GARCH, the EC-OLS, and the OLS models. The estimation is based on the initial sample and the updating procedure described above.

The picture that emerges is very much the same as within-sample. The smallest variance is achieved when hedge ratios from the full EC-GARCH model are used. The average percentage reduction in variance is considerably greater in the out-of-sample experiment. Specifically, the average reduction is 75.54% when the alternative is the unhedged position, 20.60% when the alternative is the traditional OLS, and 20.20% when the alternative is the EC-OLS. These ex ante reductions in variance are much greater than those that have been reported in the literature thus far.

Measuring Economic Significance

While our empirical findings illustrate very clearly the comparative advantage of the EC-GARCH, it is still necessary to investigate whether the percentage risk reduction is economically significant. As in Kroner and Sultan [1993], we will consider the risk reduction to be economically significant if it improves the average utility, net of transaction costs, for investors with a mean-variance utility function.

Within the mean-variance framework, the hedger's objective is to maximize the expected utility function of the form: $EU(r_h) = E(r_h) - \lambda Var(r_h)$, where r_h is the return from the hedged portfolio, and λ is the degree of risk aversion. Assuming that λ is equal to 4 and the expected return on the hedged portfolio is zero, the utility from hedging will be $-y - 4Var(r_h)$, where $-y$ are the percentage transaction costs.[7]

A typical round-trip (one buy and one sell) transaction costs $10 to $15 for an institutional investor and $25 to $50 for a retail investor, implying a minimum percentage transaction cost of 0.001% and a maximum of 0.005%. Thus, if the hedger invested in the classical hedge portfolio for the 7.5% coupon, the average utility would be $U(r_h) = -4(0.0284) = -0.1136$ per day. Had the hedger invested in the EC-GARCH hedge, the average utility would have been $U(rh) = -y - 4(0.0264) = -y - 0.1056$. That is, the investor's utility increases by $(-y + 0.008)$ with use of the dynamic EC-GARCH hedge. As a result, the dynamic hedge will be superior to the static hedge whenever $y < 0.008$.

Given that the percentage transaction costs range from a minimum of 0.001% to a maximum of 0.005%, it can be concluded that use of the dynamic hedge would produce substantial economic benefits for institutional and retail investors with a mean-variance utility function.

Our analysis is likely to understate the economic benefits of the dynamic hedge because it assumes that the investor rebalances the portfolio daily. It is more plausible to assume that a mean-variance investor will choose to rebalance only if the expected utility gains from rebalancing more than offset transaction costs.

EXHIBIT 6
Variance Comparisons of Out-of-Sample Hedging Effectiveness

Type of Hedge	FNMA 7.5	FNMA 8	FNMA 8.5	FNMA 9	FNMA 9.5	FNMA 10	Average
Panel A. Portfolio Variances							
EC-GARCH	0.0140	0.0108	0.0105	0.0118	0.0098	0.0110	
GARCH	0.0139	0.0111	0.0105	0.0118	0.099	0.0110	
EC-OLS	0.0141	0.0135	0.0153	0.0174	0.0145	0.0115	
OLS	0.0142	0.0136	0.0154	0.0174	0.0145	0.0116	
Unhedged	0.1079	0.0830	0.0627	0.0485	0.0330	0.0220	
Panel B. Percentage Variance Improvement of Conditional Hedge Compared to:							
GARCH	−0.72%	2.70%	0.0%	0.0%	1.01%	0.0%	0.49%
EC-OLS	0.71%	20.0%	31.37%	32.18%	32.41%	4.55%	20.20%
OLS	1.41%	20.59%	31.82%	32.18%	32.41%	5.17%	20.59%
Unhedged	87.03%	86.99%	83.25%	75.67%	70.30%	50.0%	75.74%

Exhibit 7

Comparisons of Full Sample Within-Sample Hedging Effectiveness n Total Expected Utility Comparisons

Hedge	Trans Cost	FNMA 7.5	FNMA 8	FNMA 8.5	FNMA 9	FNMA 9.5	FNMA 10
EC-GARCH	y = 0.005	–84.51 (753)	–72.89 (587)	–58.76 (567)	–51.07 (539)	–43.09 (442)	–81.32 (450)
GARCH	y = 0.005	–85.98 (753)	–73.58 (586)	–59.20 (565)	–51.25 (539)	–43.26 (444)	–81.82 (460)
EC-OLS		–83.02 (0)	–77.21 (0)	–63.44 (0)	–51.58 (0)	–43.75 (0)	–87.75 (0)
OLS		–84.76 (0)	–78.53 (0)	–64.53 (0)	–52.35 (0)	–44.44 (0)	–102.16 (0)

Number of portfolio rebalancings in parentheses.

Using this assumption, several within-sample and out-of-sample simulations were conducted.

The total expected utilities along with the number of times the hedge would have been rebalanced are reported in Exhibit 7 and 8 for all six coupons. With minor exceptions (the 7.5% FNMA within-sample and the 9% FNMA out-of-sample), the dynamic hedging strategies are superior to the traditional ones.

Information Content of Time-Varying Hedge Ratios

The valuation and hedging of MBS is clearly more difficult than that of other fixed-income securities (see also Breeden [1994]). The main difficulty is due to the uncertainty surrounding the amount as well as the timing of the periodic principal and interest cash flows. The source of this uncertainty is the prepayment behavior of the homeowner. In general, as interest rates fall, homeowners have an incentive to refinance (prepay). It is essential that the hedge ratio subsume the prepayment risk and, in general, all relevant information.

When market rates (i.e., mortgage commitment rates) are higher than the mortgage coupon rates, the prepayment option is out of the money. In such a case, the returns of an MBS will be highly correlated with the returns on a ten-year Treasury futures. Putting it differently, during periods of rising market rates, the correlation between an outstanding MBS and the ten-year Treasury futures will be very high since both the MBS and the Treasury futures have similar characteristics; that is, there are both convex functions of the market rate. This high correlation between the two financial assets must be impounded in the time-varying hedge ratios, causing them to be relatively higher than usual.

Conversely, whenever market rates are lower than mortgage coupon rates, the prepayment option gains in value. The implication is that the value of the MBS does not rise as fast as that of the ten-year Treasury futures. As a result, the correlation between the MBS and the ten-year Treasury futures declines.[8] Consequently, if the hedge ratios are lower during falling interest rate periods, we can say that the prepayment risk is impounded in the time-varying hedge ratio. That is, during periods

Exhibit 8

Comparisons of Holdout Sample Out-of-Sample Hedging Effectiveness Total Expected Utility Comparisons

Hedge	Trans Cost	FNMA 7.5	FNMA 8	FNMA 8.5	FNMA 9	FNMA 9.5	FNMA 10
EC-GARCH	y = 0.005	–15.80 (4)	–11.77 (38)	–14.66 (71)	–16.05 (139)	–11.17 (107)	–10.43 (25)
GARCH	y = 0.005	–17.96 (0)	–15.79 (97)	–14.55 (72)	–15.73 (120)	–11.78 (88)	–12.67 (51)
EC-OLS	–23.99 (0)	–22.31 (0)	–18.09 (0)	–14.75 (0)	–11.89 (0)	–27.35 (0)	
OLS	–24.36 (0)	–22.58 (0)	–18.33 (0)	–14.51 (0)	–12.06 (0)	–26.98 (0)	

Number of portfolio rebalancings in parentheses.

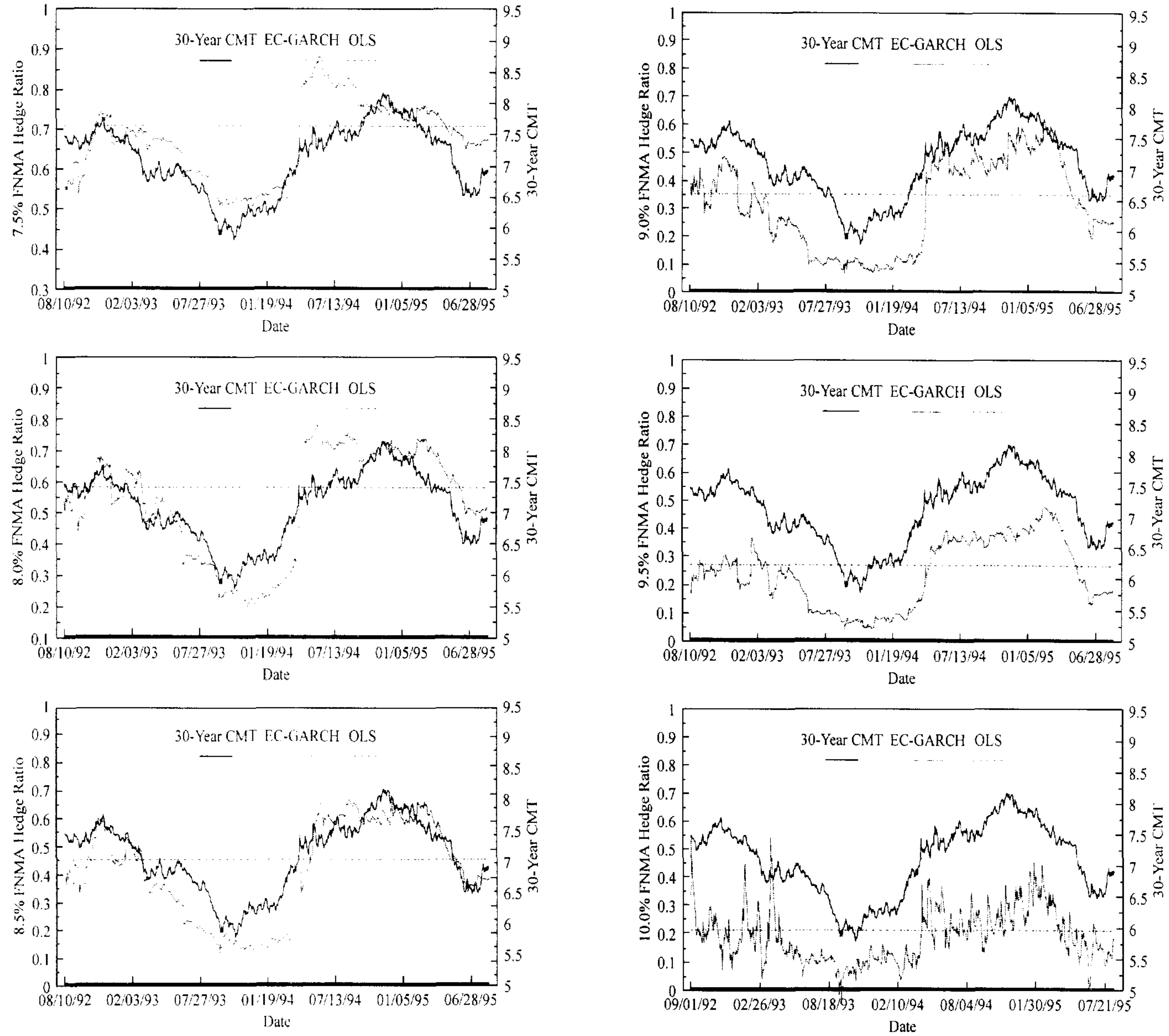
of falling market rates, an effective hedging strategy must subsume the impending increase in prepayments and thus must readjust hedge ratios downward.

Exhibit 9 shows that, indeed, there is a strong positive association between EC-GARCH hedge ratios and thirty-year Treasury rates. This constitutes proof that the dynamic hedging strategy subsumes the prepayment risk that is linked to the market rate.

To investigate the relationship between the time-varying hedge ratios and market rates formally, we test for cointegration between the estimated hedge ratios and the thirty-year Treasury rate using the Engle-Granger approach.[9] Exhibit 10, Panel A, presents the cointegrating regressions and the estimated Engle-Granger (EG) statistics. With the exception of the 7.5% coupon, the estimated EG statistics are below the 5% critical value, sug-

EXHIBIT 9

Within-Sample Hedge Ratios for Thirty-Year FNMA Along with Thirty-Tear Treasury Rates



gesting that the estimated dynamic hedge ratios and the thirty-year Treasury bond rate are cointegrated; i.e., they follow a long-term equilibrium relationship.

Panel B presents the error correction representation of hedge ratio changes. As can be seen, the coefficient for the error correction term is in all cases negative and statistically significant, suggesting that past deviations from the long-term equilibrium relationship determine current changes in the dynamic hedge ratio.

Overall, these findings show that although time-varying hedge ratios are based solely on historical information — past MBS returns and ten-year Treasury note futures returns — they subsume the prepayment risk due to changes in market rates. This is an especially appealing feature of the dynamic EC-GARCH hedges because the large losses of 1985 and 1986 were sustained by institutions that used primarily static regression-based hedges. As interest rates fell in those years, the losses in short futures positions far outpaced the gains in long MBS positions. Use of the dynamic EC-GARCH-based hedges would have dictated a much smaller hedge ratio for periods of falling market rates so that losses would have been curtailed (see Exhibit 5).

IV. CONCLUSIONS

We have proposed the use of dynamic cross-hedge ratios for mortgage-backed securities. Using an error correction model with innovations following a bivariate GARCH process (EC-GARCH), dynamic hedge ratios are calculated, and hedged portfolios are formed. The performance of these hedge ratios is evaluated on the basis of risk reduction as well as expected utility maximization.

The variance/covariance matrix of the joint distribution of the returns of FNMA MBS and ten-year Treasury futures is found to be time-dependent and can be described by a bivariate error correction GARCH model (EC-GARCH). Within-sample and out-of-sample comparisons of hedging effectiveness show that the dynamically estimated hedge ratios provide superior risk reduction than the traditional static hedge ratios. Expected utility-based simulations attest to the superiority of the dynamic hedge ratios. An interesting aspect of the proposed dynamic hedging strategy is that the estimated time-varying hedge ratios subsume information on prepayment risk even though they are based only on past MBS and ten-year Treasury note futures.

EXHIBIT 10
Information Content of Hedge Ratios

	FNMA 7.5	FNMA 8.0	FNMA 8.5	FNMA 9.0	FNMA 9.5	FNMA 10.0
Panel A: Cointegrating Regressions						
δ_0	−0.2956	−1.4203	−1.5672	−1.4927	−1.1118	−0.6266
	(0.0250)*	(0.0382)*	(0.0339)*	(0.0322)*	(0.0251)*	(0.0297)*
δ_1	0.1378	0.2762	0.2797	0.2553	0.1914	0.1153
	(0.0035)*	(0.0054)*	(0.0048)*	(0.0045)*	(0.0035)*	(0.0042)*
R^2	0.68	0.79	0.83	0.82	0.81	0.52
EG	−2.97	−3.46	−3.49	−4.42	−3.84	−7.76
Panel B: Error Correction Representation						
β_0	0.0001	0.0000	−0.0001	−0.0001	0.0000	0.0000
	(0.0003)	(0.0006)	(0.0005)	(0.0006)	(0.0004)	(0.0012)
β_{ec}	−0.0163	−0.0256	−0.0235	−0.0347	−0.0252	−0.1396
	(0.0060)*	(0.0069)*	(0.0065)*	(0.0093)*	(0.0075)*	(0.0192)*
β_1	0.0322	0.0583	0.0442	0.0588	0.0466	0.1250
	(0.0062)*	(0.0110)*	(0.0091)*	(0.0124)	(0.0078)*	(0.0237)*
R^2	0.0477	0.0630	0.0553	0.0546	0.0681	0.1115
D.W.	1.8336	1.9770	2.1218	2.1112	1.9745	2.0071

Significant at the 5% level at least.

ENDNOTES

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[1]The price of a MBS at any time can be viewed as the price of a non-callable loan minus the price of the prepayment option.

[2]The Chicago Board of Trade (CBOT) introduced the Government National Mortgage Association Collateralized Depository Receipt (FNMA CDR) in 1975. After a few years of popularity and success, investors' interest and trading volume fell substantially. The contract was finally withdrawn in 1987. Johnson and McConnell [1989] maintain that the failure of the contract was due to its flexible delivery options, which in turn reduced its hedging effectiveness. On June 1989, the CBOT introduced the mortgage-backed futures contract (MBF). To support the new contract, the exchange introduced options trading on the MBF. The MBF contract along with the options contract were withdrawn in 1992. Nothaft, Lekkas, and Wang [1995] argue that limited market liquidity along with the existence of good hedging instruments such as Treasury bond and note futures contributed to the demise of the MBF contract.

[3]Here and in all subsequent hypotheses testing, the significance level is set at 5% for the sake of simplicity and uniformity.

[4]Augmented Dickey-Fuller tests are based on the regression model:

$$X_{i,t} = a_0 + \rho X_{i,t-1} + \sum_{s=1}^{k} a_s \Delta X_{i,t-s} + u_t$$

where $X_{i,t}$ is the series to be tested for a unit root. The null hypothesis is $\rho = 1$that and the stationary alternative is that $\rho < 1$. The 5% critical values are −1.95 if the regression is estimated without constant and time trend, −2.86 if the regression is estimated without time trend, and −3.41 if the regression is estimated with constant and time trend.

[5]The Ljung-Box statistic for N lags is a χ^2 statistic and is calculated using the formula $LB(N) = T(T+2)\sum_{j=1}^{N} X_j^2/(T-j)$, where X_j is the j-th order sample autocorrelation of the series under study, and T is the sample size.

[6]The use of price differences assumes that the underlying distribution is the normal, while, the use of logarithmic differences implies that the underlying distribution is the log-normal.

[7]This assumption on λ is in line with most empirical studies in the literature. See, for example, Grossman and Shiller [1981], and Kroner and Sultan [1993].

[8]In the language of the trade, for low market rates the price-interest rate relationship turns "negatively convex" for the MBS, while, the normal convex relationship is maintained for the ten-year Treasury futures price.

[9]Cointegration tests can be applied only to two (or more) variables that have a unit root in their univariate representation. We saw earlier that the estimated dynamic hedge ratios do have a unit root in their univariate representation. The estimated ADF statistic for the thirty-year Treasury bond rate is −1.45, suggesting that the thirty-year rate has a unit root. Thus, cointegration tests between each hedge ratio and the three-year rate are legitimate.

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